Lesson Plan – January 23rd

Goal: Students will be able to derive the formula for the cardinality of the union of three sets.

# Segue to sets (8 mins)

Consider question (4a) from the previous worksheet. Now instead of thinking about each stack as a separate entity, let’s think about it in terms of the set of all bottom chips and the set of all top chips Then, the set of all possible stacks is . Thus, when we think about the multiplicative principle we are really asking for the cardinality of the Cartesian product of these two sets.

What about the additive principle? It wouldn’t make sense to just add the elements in the sets and , so maybe we should think about defining some new sets.

and .

Then the additive principle is just thinking about because you are combining the number of elements from each set, and there is no overlap (at least in this case).

What would happen if there is an overlap? – this is what we will be exploring today.

# Overlap with 2 sets (8 mins)

Let’s consider two different sets

If there was no overlap between the sets, I would take and add it to to get that . However, because there is overlap, . In fact, we came to the answer of by adding and then subtracting out what those two sets had in common, namely . Therefore, if we wanted to know the cardinality of *any* two sets we could take

Show this using a venn diagram like the one below

So, how could we think about a formula for 3 sets? Will it be as straightforward as the formula for two sets? This is something I would like you to think about when working on the following problem.

# Pie Problem (15 mins)

Some probing questions:

If someone likes pie A does it mean that they can’t like pie B?

If someone likes pies A and B does it mean that they can’t like pie C?

What type of visual aid would best serve our purposes?

Using the pie problem as an example, could you generalize our rule from before that was for 2 sets?